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LETTER TO THE EDITOR

A photon rest mass and intergalactic magnetic fields

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Abstract. Conditions for the existence and stability of the currents which would be required to support an intergalactic magnetic field are examined to see what upper limit could be obtained for the photon rest mass if the existence of such a field is established.

The effect of a nonzero photon rest mass m can be incorporated into electrodynamics by replacing Maxwell's equations with their simplest relativistic generalization, the Proca equations; these lead to $\Box^2 \boldsymbol{A} - \mu^2 \boldsymbol{A} = -(4\pi/c)\boldsymbol{j}$ where $\boldsymbol{A}$ is the vector potential, which is related to the magnetic field $\boldsymbol{H}$ by $\boldsymbol{H} = \nabla \times \boldsymbol{A}$, μ^{-1} is the reduced Compton wavelength of the photon, c is the maxwellian speed of light in free space and $\boldsymbol{j}$ is the current density; A and H will denote $|\boldsymbol{A}|$ and $|\boldsymbol{H}|$. Take $|\Box^2 \boldsymbol{A}| \simeq AL_1{}^{-2}$, so that L_1 is a characteristic length over which $\boldsymbol{A}$ varies significantly. If $L_1{}^2 \gg \mu^{-2}$, then the above equation for $\boldsymbol{A}$ leads to

$$\mu^2 \lesssim \frac{4\pi}{c}\frac{j_m}{A} \tag{1}$$

where j_m denotes the magnitude of the maximum current density that could exist in the plasma. Take $A \simeq HL_2$, so that L_2 is another characteristic length over which $\boldsymbol{A}$ varies significantly. For magnetohydrodynamic conditions, both L_1 and L_2 will usually be the smallest dimension of a quasi-uniform magnetic field.

Goldhaber and Nieto (1971) suggested that since there is a maximum current density that a plasma can support, knowledge of magnetic fields in the Galaxy can be used to obtain an upper limit on m. In this way, it has been shown (Goldhaber and Nieto 1971, Byrne and Burman 1973) that $\mu^{-1} \gtrsim 3 \times 10^{14}$ cm, corresponding to $m \lesssim 10^{-52}$ g.

Evidence that Faraday rotation occurs in intergalactic space has been presented (Sofue *et al* 1968, Kawabata *et al* 1969, Reinhardt and Thiel 1970, Fujimoto *et al* 1971). Also, it has been claimed (Reinhardt 1971, Thiel 1972) that the orientations of spiral galaxies and radio sources in the sky can be explained by the existence of an intergalactic magnetic field. Other evidence for the existence of such a field has been discussed by Piddington (1972).

Kawabata *et al* (1969) deduced that, if the Hubble constant is 100 km s^{-1} Mpc^{-1}, then $nH \simeq 2 \times 10^{-14}$ G cm^{-3}, n being the electron number density in intergalactic space. They found that the field extends to a distance corresponding to a red shift z of at least 1·4, so that a plausible estimate of L_2 would be at least 10^{27} cm. According to Fujimoto *et al* (1971) the field extends to at least $z = 2$. Arp (1971) has re-analysed the

Faraday rotation data using his interpretation of quasar red shifts in which they are not entirely due to the cosmic expansion. He found that $nH \sim 10^{-13}$ to 10^{-12} G cm^{-3}. In Arp's model, the magnetic field could be ordered over distances corresponding to groups or clusters of galaxies, so that $L_2 \sim 10^{24}$ cm.

It has been claimed (Brecher and Blumenthal 1970, Reinhardt 1972, Rees and Reinhardt 1972, Mitton and Reinhardt 1972) that the Faraday rotation data show no clear evidence for an intergalactic magnetic field. According to Rees and Reinhardt (1972) it can be deduced that $nH \lesssim 10^{-14}$ G cm^{-3}.

Stix (1962) has discussed instabilities that occur in a plasma when the electron drift speed V exceeds the phase speeds of some wave types. The magnetosonic overstability arises when V exceeds the Alfvén speed V_A; the same overstability will occur, owing to excitation of the ion cyclotron wave, when $V \gtrsim (V_A{}^2 U_i)^{1/3}$ where U_i is the ion thermal speed. The observed (Stix 1962, Alfvén 1968) critical value of V in laboratory plasmas is around U_i. If V_m denotes the maximum electron drift speed that the plasma can support stably, then $j_m \lesssim neV_m$ where e is the magnitude of the electronic charge. Since $V_A = H/(4\pi\rho)^{1/2}$ where ρ is the mass density of the medium, $j_m/A \lesssim ne/L_2(4\pi\rho)^{1/2}$ when $V_m = V_A$: the right-hand side of (1) is independent of H in this case, but is not independent of $\boldsymbol{H}$ since the geometry of $\boldsymbol{H}$ determines L_2.

A cosmological model in which the cosmic medium is highly ionized hydrogen, with, at the present epoch, $n \sim 10^{-5}$ cm^{-3} and the electron temperature T around 3×10^5 K, appears to be compatible with observations (Sciama 1971, see p 191) ; near-by intergalactic gas might be less highly ionized (Sciama 1971, see p 191).

Since V_m cannot exceed c, taking $n \sim 10^{-5}$ cm^{-3}, $H \sim 10^{-9}$ G and $L_2 \sim 10^{27}$ cm, (1) shows that $\mu \lesssim 3\times10^{-16}$ cm^{-1}. Taking $V_m = U_i$ with $U_i \sim 10^7$ cm s^{-1} shows that $\mu \lesssim 3\times10^{-18}$ cm^{-1}. Taking $V_m = V_A$ with $V_A \sim 10^5$ cm s^{-1} shows that $\mu \lesssim 3\times10^{-19}$ cm^{-1}.

A maximum value for the current density in intergalactic space can be obtained in another way: the rate of Joule heating $\boldsymbol{j}^2/\sigma$, where σ is the conductivity, cannot exceed the rate Λ at which thermal energy is lost by all processes; hence $j_m \lesssim (\sigma\Lambda)^{1/2}$. Let $\Lambda = \Lambda_r + \Lambda_e$ where Λ_r and Λ_e refer to radiative cooling and cooling arising through the expansion of the universe, respectively. In their treatment of the thermal history of the intergalactic medium, Ginzburg and Ozernoi (1966) included cooling by free–free and free–bound collisions in ionized hydrogen; these contribute $10^{-27}n^2T^{1/2}$ erg s^{-1} cm^{-3} and $5\times10^{-22}n^2T^{-1/2}$ erg s^{-1} cm^{-3}, respectively, to Λ_r, T being the temperature. They showed that heavy elements will have negligible effect on Λ_r. Thus if $n \sim 10^{-5}$ cm^{-3} and $T \sim 3\times10^5$ K, then $\Lambda_r \sim 10^{-34}$ erg s^{-1} cm^{-3}. Weymann (1967) extended the calculations to include the effects of line radiation and the presence of helium. For $n \sim 10^{-5}$ cm^{-3} and $T \sim 3\times10^5$ K his figure 1 shows that $\Lambda_r \sim 3\times10^{-34}$ erg s^{-1} cm^{-3}. According to Ginzburg and Ozernoi (1966), provided that the density ρ of the intergalactic medium is near the cosmologically critical value, $\Lambda_e \simeq (2R/M)\times(\rho_0{}^{1/2}t)^{-1}\rho^{3/2}T$ where R is the gas constant, M is the molecular weight of the intergalactic medium and ρ_0 is its density at epoch t. Taking $M = \frac{1}{2}$ and assuming that when $t \sim 3\times10^{17}$ s the density is around 10^{-29} g cm^{-3}, it follows that $\Lambda_e \sim 3\times10^5\rho^{3/2}T$ erg s^{-1} cm^{-3}. If $\rho \sim 10^{-29}$ g cm^{-3} and $T \sim 3\times10^5$ K at the present epoch, then $\Lambda_e \sim 3\times10^{-33}$ erg s^{-1} cm^{-3}. Thus, for $T \sim 3\times10^5$ K, $\Lambda \sim 3\times10^{-33}$ erg s^{-1} cm^{-3}, while the standard formula for the conductivity shows that $\sigma \sim 3\times10^{14}$ s. Taking these values together with $H \sim 10^{-9}$ G and $L_2 \sim 10^{27}$ cm gives $\mu \lesssim 10^{-18}$ cm^{-1}.

If Arp's values are used with $L_2 \sim 10^{24}$ cm, together with $n \sim 10^{-5}$ cm^{-3} so that $H \sim 10^{-3}$ to 10^{-7} G and $V_A \sim 10^6$ to 10^7 cm s^{-1}, and if $T \sim 3\times10^5$ K, then these upper limits on μ are each increased by a factor of between 3 and 10, except that obtained from putting $V_m = V_A$, which is increased by a factor of 30.

According to a recent review article (Field 1972), T could be as low as 10^4 K or as high as 3×10^8 K. Thus the limits obtained above from putting $V_m = U_i$ could be changed by a factor of roughly three either way. Reference to Weymann's figure 1 and to the above formula for Λ_e shows that if $T \gtrsim 10^4$ K, then $\Lambda_e > \Lambda_r$ and the limit obtained from the energy dissipation argument varies as $T^{5/8}$. If $T \sim 3\times10^8$ K, then the limit is increased over the above value by a factor of roughly 10^2; if $T \sim 10^4$ K, then the limit is reduced by a factor of roughly ten.

For a fixed value of nH, the limits on μ obtained from putting $V_m = U_i$ are proportional to n while those obtained from putting $V_m = V_A$ are proportional to $n^{1/4}$. If n is significantly less than the value 10^{-5} cm^{-3}, then, for fixed nH, the limits obtained above are too large; for example if $n \sim 10^{-7}$ cm^{-3}, then limits obtained from putting $V_m = U_i$ are improved by two orders of magnitude while limits obtained from putting $V_m = V_A$ are improved by factors of about three.

Williams and Park (1971) introduced a method for obtaining a limit on m which is based on the dissipation of magnetic flux in the Galaxy; it is found that $\mu \lesssim (4\pi\sigma/c^2\tau)^{1/2}$ where τ is the decay time for magnetic flux (Williams and Park 1971, Byrne and Burman 1972). Applying this formula to an intergalactic field with $\tau \gtrsim 10^{10}$ year and taking $T \sim 10^4$ K and $T \sim 3\times10^8$ K corresponding, for a fully ionized medium, to $\sigma \sim 3\times10^{12}$ s and $\sigma \sim 10^{19}$ s, leads to $\mu \lesssim 3\times10^{-13}$ cm^{-1} and $\mu \lesssim 10^{-9}$ cm^{-1}, respectively. The former value is two orders of magnitude better than the limit which has been obtained from the galactic magnetic field by this method (Byrne and Burman 1972), but is not more than one order of magnitude better than the limit which could be obtained from the galactic magnetic field by this method with improved knowledge of the interstellar medium; the latter value, 10^{-9} cm^{-1}, is an order of magnitude poorer than that obtained from the geomagnetic field (Goldhaber and Nieto 1968, 1971). The limit obtained by this method is proportional to $T^{3/4}$ and depends only very slightly on n.

Lyttleton and Bondi (1959, 1960), Hoyle (1960) and Bondi (1961), in their discussions of the effects of a possible general charge imbalance in the Universe, developed an electrodynamics which leads, in flat space–time, to the wave equation $\Box^2 A - \lambda A = -(4\pi/c)(j - \lambda^{-1}\nabla q)$; here λ is a negative constant with dimensions (length)$^{-2}$ and q is the rate of creation of charge per unit volume. If q is regarded as being constant in space, then the above wave equation reduces to $\Box^2 A - \lambda A = -(4\pi/c)j$, and the arguments developed in this paper provide the same information on $(-\lambda)^{1/2}$ as on μ. This remark applies, in particular, to Hoyle's version of the theory, which is a form of the steady-state cosmology in which both matter and antimatter are created, at equal rates, so that $q = 0$. In that model, the process of separation of matter and antimatter produces an intergalactic magnetic field. Hoyle's equations, together with values claimed by Kawabata *et al* (1969) and by Arp (1971) for nH, show (Burman 1971) that $(-\lambda)^{-1/2} \gtrsim 10^{22}$ cm if the fraction of material separated into matter regions and antimatter regions is of order one.

The considerations presented here indicate that if the existence of an intergalactic magnetic field is established, then it should be possible to obtain an upper limit on the photon rest mass around three or four orders of magnitude less than the limit obtained from similar considerations (Goldhaber and Nieto 1971, Byrne and Burman 1973) for the galactic magnetic field, and around eight orders of magnitude less than the upper limit of 4×10^{-48} g obtained by Goldhaber and Nieto (1968, 1971) from geomagnetic data. The limit that could be obtained is not very strongly dependent on the temperature of the intergalactic medium.

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